

The Most Powerful Test of Independence in $W \times K$ Contingency Tables for Ordinal Data Using Monte Carlo Simulations

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Info Articles

Keywords:

Tests of Independence, Power Analysis, Contingency Table, Ordinal data, Monte Carlo Simulations

Abstract

This study aims to analyze the performance of tests of independence for ordinal data. Tests of independence is one of the most frequently used statistical tools in econometrics. Researchers are often interested about the independence of variable summarized in Contingency Tables. Many tests are available in literature to test independence in Contingency Tables, however, there is no clarity about the choice of tests which are incapable to provide comparison of large number of tests. A central problem and question facing researcher is to decide which tests of independence is most powerful for ordinal data. Most of the studies make pairwise comparison of tests and such studies are unable to guide about optimal test among a wide set of tests. This study used Monte Carlo Simulations (MCS) and compares seven popular tests of independence for ordinal data namely, Spearman ρ coefficient of correlation, Kendall's $\tau - a$, Kendall's $\tau - b$, Kendall's $\tau - c$ coefficient, Goodman and Kruskal γ , Sumer's D and Novel Φ_k (Φ_k). For ordinal data the stringency criteria cannot be applied to compare tests. Therefore, the comparison was made on the basis of power analysis and our MCS concludes based on solid estimations using power criterion that the most powerful test is Novel Φ_k in $w \times k$ CT's for ordinal data.

INTRODUCTION

Economists and others researchers often use to test the independence of variables organized in the form of Contingency Tables (CTs). Contingency table's information is used in a wide range in numerous fields, including social sciences, (Wickens 1969), Genetics (El Galta, Stijnen et al. 2008, Dickhaus, Straßburger et al. 2012), Demographics, Cung (2013) and in Psychology (Iossifova et al., 2013).

The categorical data in CTs can further classified in nominal and ordinal data. There are large numbers of tests available in literature which can be applied for testing independence. However, the literature is generally silent about the appropriate choice of test. There are few studies compares tests of independence. Most of these studies makes a pairwise comparison and do not provide guidance for choice among large class of tests.

Thus, the literature is silent and no consensus has been developed on the most powerful test of independence in $w \times k$ CTs. This study makes comparison of a large numbers of tests of independence for ordinal data. Consider the data types described in Table 1.

Table 1. Typical $W \times K$ CT for Variable X and Y

Variable X	Variable Y				Total
	Y_1	Y_2	...	Y_K	
X_1	n_{11}	n_{12}	...	n_{1k}	$n_{1.}$
X_2	n_{21}	n_{22}	...	n_{2k}	$n_{2.}$
..	...	...	...	...	...
X_W	n_{w1}	n_{w2}	...	n_{wk}	$n_{w.}$
Total	$n_{.1}$	$n_{.2}$	...	$n_{.k}$	N

Table 1 elaborate that if we have “n” draws having two variables ‘X’ and ‘Y’; each character is having certain categories. n_{11} , is the numbers of draws having category 1 in ‘X’ variable and category 1 in ‘Y’ variable. Similarly, n_{wk} is number of draws having category ‘w’ in ‘X’ variable and category ‘k’ for ‘Y’ variable such that the categories of ‘X’ and ‘Y’ can have a natural ordering and in that case, data would be termed as ordinal data. While sometime the categories do not have any order and are known is nominal data.

The marginal sums in the $w \times k$ CT would be:

$$n_{1.} = \sum_{j=1}^k n_{1j}, n_{2.} = \sum_{j=1}^k n_{2j}, \dots, n_{w.} = \sum_{j=1}^k n_{wj} \quad (1.1)$$

$$n_{.1} = \sum_{i=1}^w n_{i1}, n_{.2} = \sum_{i=1}^w n_{i2}, \dots, n_{.k} = \sum_{i=1}^w n_{ik} \quad (1.2)$$

The value “N” is the sum of all the counts of the $w \times k$ CT.

$$N = \sum_{i=1}^w n_{i.} = \sum_{j=1}^k n_{.j} = \sum_{i=1}^w \sum_{j=1}^k n_{ij} \quad (1.3)$$

Suppose each entry of Table 1 is divided by ‘n’ such that the Table 2 takes following forms.

Table 1. $W \times K$ Contingency Table for Observed Frequencies Variable X and Y

Variable X	Variable Y				Total
	Y_1	Y_2	...	Y_j	

$\mathbf{X}_1$	$n_{11/n} = p_{11}$	$n_{12/n} = p_{12}$	...	$n_{1j/n} = p_{1j}$	$p_{1.}$
$\mathbf{X}_2$	$n_{21/n} = p_{21}$	$n_{22/n} = p_{22}$	...	p_{2j}	$p_{2.}$
...	...	...	...	...	...
$\mathbf{X}_i$	$n_{i1/n} = p_{w1}$	$n_{i2/n} = p_{w2}$	...	p_{ij}	$p_{i.}$
Total	$p_{.1}$	$p_{.2}$	...	$p_{.j}$	1

Table 2 shows that p_{11} represents proportions of draws having category 1 in 'X' and category 1 in 'Y'. Whereas, p_{ij} represents category 'i' in variable 'X' and category 'j' in variable 'Y' and the term is as follows;

$$\sum_{i=1}^w \sum_{j=1}^k p_{ij} = 1 \quad (1.4)$$

The actual probabilities are shown in Table 3 as follows:

Table 3. Theoretical Distribution of Variable X and Y in $W \times K$ CTs

	'Y' Variable				Marginal Probability
	(π_{11})	(π_{12})	-	(π_{2j})	$\pi_{1.}$
'X' Variable		(π_{22})	-	(π_{2j})	$\pi_{2.}$
	-	-	-	-	-
	-	-	-	-	-
	(π_{i1})	(π_{i2})	...	(π_{ij})	$\pi_{i.}$
Marginal Probability	$\pi_{1.}$	$\pi_{2.}$		$\pi_{j.}$	1

Table 2 shows that p_{11} represents proportions of draws having category 1 in 'X' and category 1 in 'Y'. Whereas, p_{ij} represents category 'i' in variable 'X' and category 'j' in variable 'Y' and the term is as follow;

$$\sum_{i=1}^w \sum_{j=1}^k p_{ij} = 1 \quad (1.4)$$

Table 2 are random draws; the actual probability could be different from observed proportions. The actual probabilities are shown in Table 3 as follow:

If, $\frac{n_{ij}}{n}$ (Table 2) gives observed frequency for a particular cell. Then π_{ij} would be the theoretical probabilities associated with the cell. Researchers are mostly interested in the variables, especially when looking at the CTs to see if there is a relationship between variables, whether they are independent or not.

Therefore, condition for independence can be written as follows: Suppose we have equation 1.5.

$$\pi_{.j} = \sum_{i=1}^w \pi_{ij} \quad (1.5)$$

Thus, the condition for independence is the probability of any cell in the CTs equal to the product of row and column probability of the concerned cell. This condition transforms into equation 1.7.

$$\pi_{i,j} = \pi_i \times \pi_j \quad \text{For all } i, j. \quad (1.6)$$

This gives us;

$$\sum_{i=1}^w \pi_{ij} = \sum_{j=1}^k (\pi_i \pi_j) \quad (1.7)$$

Thus, the null and alternative hypothesis for independence becomes,

$$\text{Null Hypothesis : } \sum_{i=1}^w \sum_{j=1}^k (\pi_{ij} - \pi_i \pi_j)^2 = 0 \quad (1.8)$$

$$\text{Alternative Hypothesis: } \sum_{i=1}^w \sum_{j=1}^k (\pi_{ij} - \pi_i \pi_j)^2 \neq 0 \quad (1.9)$$

$$\text{For } i = 1 \dots \dots w, \quad j = 1 \dots \dots k.$$

In this context, this study is aimed to evaluate the performance of the independence test for a variety of Data Generating Process (DGP) for ordinal data in CTs. We used power criterion (PC) for most powerful tests of independence for ordinal data.

LITTÉRATURE REVIEW

Several studies have been conducted on comparisons of tests of independence for categorical data particularly with respect to ordinal data. From 1900 to the present day, several tests have been invented and modified; criticism have been made on tests of independence at different time due to data assumptions, the nature of dimensions, statistical techniques, and the nature of variable types in the contingency table (CTs). Accordingly, over the time period on one hand a number of tests have been developed; while, on the other hand, the question remains that which test is the most powerful test and which test should be used for a particular type of ordinal data. In response to this specific question numerous studies have been conducted, e.g., (Simon 1978), Lawal, (Chan, Yung et al. 1998, Quessy 2009), Yenigün, Székely et al. (2011), Berry and Mielke Jr (1988), Lipsitz, Fitzmaurice et al. (2015), (Sulewski and Motyka 2015), Sulewski (2017) and (Islam and Rizwan 2020) etc. There is still no consensus on the powerful tests for ordinal data.

Limited literature exists on the tests of independence for ordinal data; However, most of these are limited in scope and do not come to the precise conclusion of finding the most powerful test of independence for ordinal data in a class of large number of tests. Charlas, (1961) discussed in his book the relative efficiency of four measure of correlation i.e., Pearson product moment correlation, Kendal τ_a, τ_b and τ_c and found that Kendal Tau is more reliable measure of correlation/test of independence. Selecting the most powerful test from the literature is a central problem in the social sciences and the primary question facing researchers is figuring out which test is to be used for available data.

Tests of Independence for Ordinal Data in $W \times K$ CTs

This study takes into account seven popular tests of independence used for ordinal data in $w \times k$ CTs which are described below;

Spearman's Rank Correlation Test (ρ)

The spearman's rank correlation test is a non-parametric test of independence / measure of strength and direction of association that exists between two variables measured as ordinal scale. It is denoted by symbol ' r_s ' and Greek letter ' ρ '. It is identical to the Pearson correlation coefficient calculated for ranks which is used to describe the strength of correlation of two variables, especially when they are qualitative nature. When the

number of observations is small, it can be used to examine the relationship between quantitative variables by prior ranking. Spearman's ρ is described by the formula;

$$\rho = r_s = 1 - \frac{6d^2}{N(N^2 - 1)} \quad (1.10)$$

where, d^2 is the measure of inversion, N - the number of observations (sample size). Based on the “ n ” of a sample taken from population, the null hypothesis is that spearman “ ρ ” coefficient is zero i.e., ($H_0 : \rho_s = 0$) against the alternative hypothesis ($H_1 : \rho_s \neq 0$) $\vee H_1: \rho_s > 0 \vee H_1: \rho_s < 0$.

The Kendall τ -a coefficient

The Kendall τ -a coefficient is used only in cases where the so-called tied (related) ranks. Linked pairs occurs when not all observations have the same values and respondents cannot be strictly ordered by the value of a given variable. A pair is said to be linked if the same value (rank) is observed for one or both variables. Relation can be due to variable ‘X’, variable ‘Y’, or both. For two-way tables, all cases falling into the same category of one variable (row or column) are related to each other.

There are five types’ pairs:

- N_c ordered pairs consistent with respect to X and Y variables;
- N_d ordered pairs inconsistent with respect to X and Y variables;
- T_x pairs related to X but ordered to Y;
- T_y pairs related to Y but ordered to X;
- T_{xy} pairs related due to both variables X and Y.

The number of all possible pairs is equal to the sum of all the types mentioned above, therefore,

$$N_c + N_d + T_x + T_y + T_{xy} = \frac{n(n-1)}{2} \quad (1.11)$$

If the difference $N_c - N_d$ is divided by the number of all possible pairs in the ‘ N ’ element set, the coefficient proposed by Kendall in the form (Kendall 1938).

$$\tau_a = \frac{N_c - N_d}{\binom{n}{2}} = \frac{2(N_c - N_d)}{n(n-1)} \quad (1.12)$$

If the empirical data is written in the form of $w \times k$ CT and the categories of this table are ordered, then

$$N_c = \sum_{i=1}^w \sum_{j=1}^k n_{ij} C_{ij} \quad (1.13)$$

$$C_{ij} = \sum_{a=1}^{i-1} \sum_{b=1}^{j-1} n_{ab} + \sum_{a=i+1}^w \sum_{b=j+1}^k n_{ab} \quad (1.14)$$

$$N_d = \sum_{i=1}^w \sum_{j=1}^k n_{ij} D_{ij} \quad (1.15)$$

$$D_{ij} = \sum_{a=1}^{i-1} \sum_{b=j+1}^k n_{ab} + \sum_{a=i+1}^w \sum_{b=1}^{j-1} n_{ab} \quad (1.16)$$

If numbers of matched pairs are denoted by N_c and unmatched pairs by N_d ; then the difference between $N_c - N_d$ is the difference between the matched pairs and unmatched pairs. In case of $N_c - N_d > 0$, the relationship is positive while in case of $N_c - N_d < 0$ the relationship is negative.

The Kendall coefficient of τ -b

The Kendall τ -b coefficient proposed by Kendall has the following computational form;

$$\tau_b = \frac{N_c - N_d}{\sqrt{(N_c - N_d + T_x)((N_c - N_d + T_y))}} \quad (1.17)$$

Its popularity is due to the fact that it takes on values close to Pearson's linear correlation coefficient, especially when the number of categories for each of the analyzed variables is not less than 5. The τ -b coefficient is symmetric, takes values from the interval, but takes extreme values only for square tables. It is the geometric mean of the two asymmetric Somer's D coefficients

$$\tau_b = \pm \sqrt{d_{yx} - d_{xy}} \quad (1.18)$$

If the empirical data is written in higher order CT then the following formula is used for computation;

$$\tau_b = \frac{N_c - N_d}{\sqrt{D_w - D_k}} \quad (1.19)$$

Kendall Stuart τ -c coefficient

The Kendall Stuart τ -c coefficient proposed by Kendall and Stuart for CTs has the following computational form stated by Kendall and Stuart, (1973);

$$\tau_c = \frac{2m(N_c - N_d)}{n^2(m - 1)} = m = \min(w, k) \quad (1.20)$$

It was designed specifically for tables and can formally take values from $(-1 \text{ to } +1)$. Interpreting its size is difficult as it is strongly dependent on the size of the table. The τ -c coefficient is symmetrical and have no proportion reduction error (PRE) interpretation.

Goodman – Kruskal Gamma (γ)

The γ coefficient proposed by Goodman and Kruskal does not take into account bonded pairs, it can be computed from the following formula, Goodman and Kruskal, (1954);

$$\gamma = \frac{N_c - N_d}{N_c + N_d} \quad (1.21)$$

This coefficient is symmetric and takes on values from the range. Values close to zero indicate that there is no or only a weak relationship between the variables, values close to $|1|$ means a strong dependence. Gamma can be used as a test of independence using a Z score where the null hypothesis is $H_1 =$ no association against the alternative hypothesis of $H_1 =$ there is association amongst the variables.

Somers'D coefficient

The Somers delta or “d” coefficient proposed by somer's taking into account bonded pairs has the form; (Somers 1962).

(Y - Dependent variable)

$$d_{y \mid x} = \frac{N_c - N_d}{N_c + N_d + T_y} \quad (1.22)$$

(X - Dependent variable)

$$d_{x \mid y} = \frac{N_c - N_d}{N_c + N_d + T_x} \quad (1.23)$$

The “d” coefficient is asymmetric; its size depends on which variable is dependent. Comparing it with the coefficient γ , it was found that it does not reach an absolute value greater than γ . It takes values from the interval. If the number of columns is greater than the number of rows, it does not get the value 1, because then there are connections due to Y. Likewise (X is a dependent variable) it does not get the value 1 when the number of rows is greater than the number of columns.

Somer's also used the following formula for symmetrical variants.

$$d_s = \frac{N_c - N_d}{N_c + N_d + 0,5(T_x + T_y)} \quad (1.24)$$

(Y - Dependent variable)

$$d_{y/x} = \frac{N_c - N_d}{D_w} \quad (1.25)$$

(X - Dependent variable)

$$d_{x/y} = \frac{N_c - N_d}{D_k} \quad (1.25)$$

$$d_s = \frac{N_c - N_d}{0,5(D_w + D_k)} \quad (1.26)$$

$$D_w = n^2 - \sum_{i=1}^w n_{i.}^2 \quad (1.27)$$

$$D_k = n^2 - \sum_{j=1}^k n_{.j}^2 \quad (1.28)$$

D_w , D_k are the coefficients, Somers's coefficient also has an interpretation in terms of PRE and is analogous to the interpretation of coefficient γ . The difference is that proportional error reduction is about predicting the ordering of unrelated pairs with respect to the independent variable. Coefficient factor d_{yx} can be interpreted as the probability that random observation 'j' ranks higher/lower and variable 'Y' when it ranks higher on variable 'X'.

Novel Φ_k (Φ_k) Correlation

The Novel Φ_k correlation is useful for assessing the association between nominal, ordinal, ratio and interval variables. This has the specialty that it does not only capture the linear association but nonlinear association as well in CTs. The calculation of correlation coefficients between paired data variables is a standard tool of analysis for every data analyst. Pearson's correlation coefficient is a de-facto standard in most fields, but by construction only works for interval variables (sometimes called continuous variables). Pearson is unsuitable for data sets with mixed variable types, e.g. where some variables are ordinal or categorical. The correlation coefficient Novel Φ_k follows a uniform treatment for interval, ordinal and categorical variables, captures non-linear dependencies, and is similar to Pearson's correlation coefficient in case of a bivariate normal input distribution.

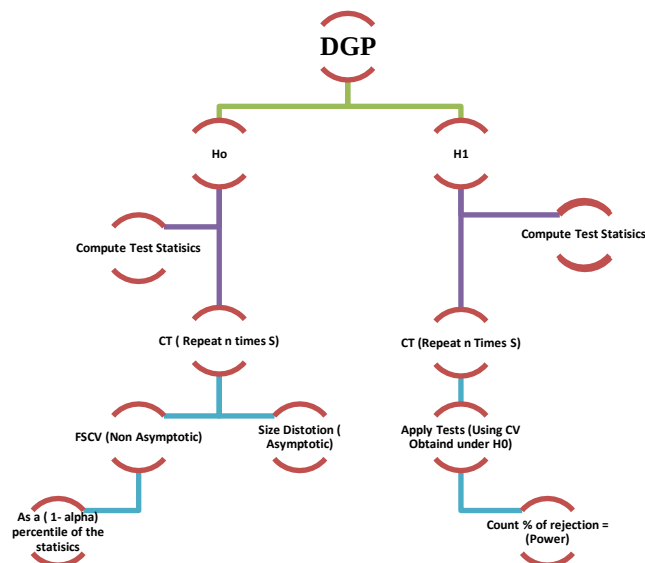
The Novel Φ_k Correlation estimator is computed as:

- A $w \times k$ CTs is created, filling of the CT for ordinal data or chosen variable pair, which contains N records, has w rows.
- Evaluate the χ^2 contingency test using the Pearson's χ^2 test statistic
- Interpret the χ^2 value as coming from results and if $\chi^2 < \chi^2_{pre}$, set $\rho = 0$.
- Else, with fixed N, w, k, χ^2 invert the function and solve numerically for rho value.
- The solution for ρ defines the correlation coefficient Novel Φ_k .

Methodology for Power analysis

Methodology for tests of independence in $w \times k$ CTs are discussed below.

Simulation Design



The core objective is to assess the performance of tests of independence for ordinal data through comparing the power of tests using the power criteria. To achieve this specific objective, the study mainly focuses on Monte Carlo Simulations (MCS). We have analyzed the performance of tests of independence using algorithms on the basis of MCS and through Power analysis comparison find out the most powerful test of independence for ordinal data in $w \times k$ CTs.

The proposed methodology consists of the following three steps:

- Data generating process (DGP)
- Methods for calculation of Finite Sample Critical Values (FSCV)
- Computation of Power

The sub objective of simulation experiment is to find out power properties of tests of independence for ordinal data. Therefore, we need a number of DGP in $w \times k$ dimensions CTs. The selection of the DGP for the MCS study is very important mostly in the comparative analysis. The tests or approaches can be compared in the same framework to recommend the superiority of one test or the weakness of another test. Fig 1.1 shows simulation design for present study.

Figure 1. Simulation design for categorical data

To achieve most powerful test of independence / measure of correlation for ordinal data. This study compares seven popular tests of independence on the basis of Power. For simulation purpose we created the following algorithm.

Let X be a random variable which can be ordered into K categories. The variable can be generated as

$$x_1 \sim U(0, k)$$

Then X becomes categorical random variable

$$X = \text{round}(x_1) \quad (1.20)$$

Let Y is another variable which is dependent on X be generated as

$$Z_1 = U(0, k_2) \quad (1.21)$$

Where k_2 is numbers of categories in Y .

$$\text{Suppose, } y = ax_1 + bz_1 \quad (1.22)$$

where $a+b = 1$,

Then

$$Y = \text{round}(y_1)$$

The equation 1.22 can give us perfectly correlated variables when $a = 1$ and $b = 0$ and perfectly independent when $a = 0$ and $b = 0$.

Thus, correlation is determined by a, b ($a+b=1$) where $a=1$ and $b=1$ then there is perfect correlation and are independent.

Finite Sample Critical Values (FSCV) and Power Analysis

All the tests of independence/Measure of correlations for ordinal data are non-parametric and critical values are calculated by simulations. Therefore, it is useless to calculate size distortion. However, power shall be calculated as described below.

Computation of Finite Sample Critical Values in CTs

The following steps are involved in calculation of finite sample critical values.

- Generate data under H_0
- Arrange data in $w \times k$ contingency tables.
- Calculate tests statistics (Selected one of seven tests taken under this study).
- Repeat 20,000' times (MCS).
- Critical Value is $(1 - \alpha)$ percentile of the tests statistics obtained.

Computation of Power in CTs

- Generate data under H_1 with pre-specified MoU.
- Arrange data in $w \times k$ contingency tables.
- Calculate test statistics, decide acceptance / rejection using FSCV.
- Repeat step a, b and c "20,000" times (MCS) and calculate power = % of rejections.

RESULTS AND DISCUSSION

The key objective of this article is to evaluate the most powerful test of independence/measure of correlation in $w \times k$ contingency table for ordinal data. The power of test is defined as the probability of rejecting null hypothesis when it is false i.e.

$$Power = P(\text{Rejecting } H_0 / H_1 \text{ is True})$$

This study used small, medium and large sample size according to the size of CTs with nominal level (α level 5%). The study used simulated critical values (SCV) computed for small medium and large numbers of samples.

We investigated power analysis of tests of independence for ordinal data and found the following results stated in Tables 4.

Table 4. Power Analysis of Tests of independence for Ordinal Data for 12×12 Contingency Table

Nominal Level (α) =5%	Strength of Correlation [SoC]										N=800
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.050	0.065	0.087	0.198	0.245	0.352	0.489	0.598	0.621	0.771	0.889
Kendall τ -a	0.050	0.058	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
Kendall τ -b	0.051	0.053	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
Kendall τ -c	0.054	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.582	0.698	0.723

Gd – Kruskal γ	0.052	0.064	0.168	0.289	0.399	0.488	0.558	0.699	0.746	0.882	0.924
Somers'd	0.052	0.063	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
Novel Φ_k	0.050	0.198	0.262	0.378	0.498	0.556	0.698	0.751	0.871	0.938	1.000

Since we know from above equations;

$$Y = aX + bZ \quad ; a + b = 1$$

where, "a" determine strength of correlation in GDP. The result of table 7 indicates that as the strength of correlation increases, the power of the tests increases as well. The results further explains that Spearman ρ , Goodman Kruskal γ and Novel Φ_k has the maximum powers at nominal level ($\alpha = 5\%$) at sample size 25. The others test Kendall τ -a, Kendall τ -a, Kendall τ -a and Somers'd have lower power in this case.

CONCLUSION

All of the above tests of independence are non-parametric and we cannot find likelihood ratios for these measures of correlations. Therefore, we cannot use stringency criteria to evaluate most stringent test of independence for ordinal data. Thus, we used power analysis techniques (PC) through simulations and compared seven measures of correlation including a novel correlation known is Novel Φ_k .

Therefore, Current study also found Kruskal Gamma and Spearman Rank correlations close to the Novel Φ_k correlation in terms of power comparison for higher order contingency table in analysis of ordinal data. The study also concludes that in terms of powerful tests for ordinal data are ranked sequentially are Novel Φ_k , Gd - Kruskal γ , Spearman ρ , Somers'd, Kendall τ -c, Kendall τ -b and Kendall τ -a.

Despite the fact that the study has some limitations, however we believe that our findings should prove beneficial for researchers and practitioner. In this regard following are some recommendations.

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